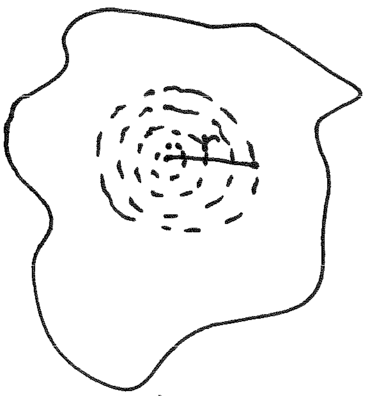
Related Rates Worksheet

1. Sheila walks to Lake Menomin and throws a rock into the lake. Since the lake is calm, ripples in the shape of concentric circles are formed on the water. If the radius of the outer ripple is increasing at a rate of 2 feet per second, at what rate is the total area of disturbed water changing when the radius is 5 feet?



know

$\frac{dr}{dt} = 2$ ft/sec.

want

$\frac{dA}{dt}$

when $r = 5$ft.

equation

$A = \pi r^2$

$\frac{d}{dt}\left[A(t) = \pi[r(t)]^2\right]$

$A'(t) = \pi(2)[r(t)]\, r'(t)$

$\frac{dA}{dt} = 2\pi r \left[\frac{dr}{dt}\right]$

when $r = 5$

$\frac{dA}{dt} = 2\pi(5)(2)$

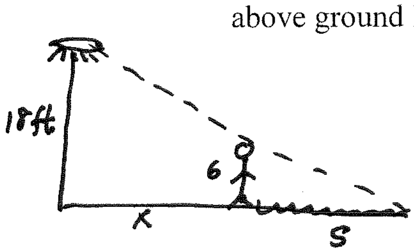
$\frac{dA}{dt} = 20\pi$ ft^2/sec

r = radius of outer ripple (ft)

A = area of disturbed water (ft^2)

t = time (seconds)

2. A person 6 feet tall walks away from a streetlight at the rate of 5 ft/s. If the light is 18 feet above ground level, how fast is the person's shadow lengthening?



s = length of shadow (ft)

x = distance from streetlight (ft)

t = time (seconds)

know

$\frac{dx}{dt} = 5$ ft/sec.

want

$\frac{ds}{dt}$ when

equation

$\frac{6}{s} = \frac{18}{x+s}$

$6(x+s) = 18s$

$6x + 6s = 18s$

$6x = 12s$

$\frac{d}{dt}\left[6[x(t)] = 12[s(t)]\right]$

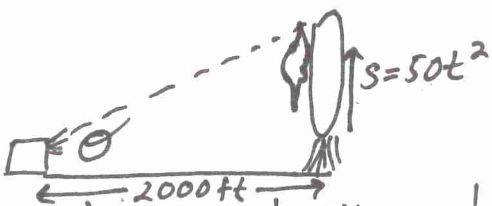
$6x'(t) = 12s'(t)$

$6\frac{dx}{dt} = 12\frac{ds}{dt}$

$\frac{6(5)}{12} = \frac{ds}{dt}$

$\frac{30}{12} = 2.5$ ft/sec. $= \frac{ds}{dt}$

3. A television camera at ground level is filming the lift-off of a space shuttle. The shuttle is rising vertically according to the position equation $s = 50t^2$, where s is measured in feet and t is measured in seconds. The camera is 2000 feet from the launch pad. Find the rate of change in the angle of elevation of the camera at 10 seconds after lift-off.



s = distance shuttle is above the ground (ft)

t = time (seconds)

θ = angle of elevation of camera (radians)

given

$$\frac{ds}{dt} = 100t$$

want

$$\frac{d\theta}{dt} \text{ when } t = 10$$

equation

$$\tan(\theta) = \frac{opp}{adj} = \frac{s}{2000}$$

$$\tan(\theta) = \frac{50t^2}{2000}$$

$$\tan(\theta) = \frac{1}{40}t^2$$

$$\frac{d}{dt}\left[\tan(\theta(t)) = \tfrac{1}{40}t^2\right]$$

$$\sec^2(\theta(t))\frac{d\theta}{dt} = 2(\tfrac{1}{40})t$$

$$\sec^2(\theta)\left[\frac{d\theta}{dt}\right] = \tfrac{1}{20}t$$

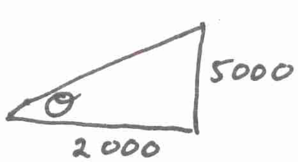
when $t = 10$

$$\sec^2(\theta)\left[\frac{d\theta}{dt}\right] = \tfrac{1}{20}(10)$$

$$\sec^2(\theta)\frac{d\theta}{dt} = \tfrac{1}{2}$$

when $t=10$

$$s = 50(10)^2 = 5000$$



$$\tan\theta = \frac{5000}{2000}$$

$$\tan\theta = \tfrac{5}{2}$$

$$\tan^{-1}(\tfrac{5}{2}) = \theta$$

$$\theta \approx 1.19$$

$$\sec^2(1.19)\frac{d\theta}{dt} = \tfrac{1}{2}$$

$$\frac{d\theta}{dt} = \frac{\tfrac{1}{2}}{\sec^2(1.19)}$$

$$\frac{d\theta}{dt} = \frac{\tfrac{1}{2}}{7.2395}$$

$$\frac{d\theta}{dt} = .069 \text{ radians/sec}$$